

Newton's Second Law of Rotation Activity

Introduction

We have thus far found many counterparts to the translational terms used throughout this text, most recently, torque, the rotational analog to force. This raises the question: Is there an analogous equation to Newton's second law, $\Sigma F = ma$, which involves torque and rotational motion? To investigate this, we start with Newton's second law for a single particle rotating around an axis and executing circular motion. Let's exert a force \vec{F} on a point mass m that is at a distance r from a pivot point ([Figure](#)).

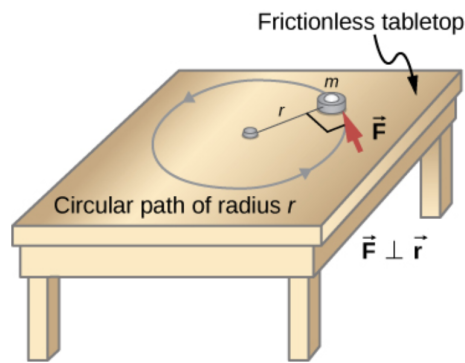


Figure 10.37 An object is supported by a horizontal frictionless table and is attached to a pivot point by a cord that supplies centripetal force. A force \vec{F} is applied to the object perpendicular to the radius r , causing it to accelerate about the pivot point. The force is perpendicular to r .

The particle is constrained to move in a circular path with fixed radius and the force is tangent to the circle. We apply Newton's second law to determine the magnitude of the acceleration, $a = F/m$, in the direction of F . Recall that the magnitude of the tangential acceleration is proportional to the magnitude of the angular acceleration by $a = r\alpha$. Substituting this expression into Newton's second law, we obtain, $F = mr\alpha$.

Multiply both sides of this equation by r , we get $rF = mr^2\alpha$.

Note that the left side of this equation is the torque about the axis of rotation, where r is the lever arm and F is the force, perpendicular to r . Recall that the rotational inertia for a point particle is $I = mr^2$ and the torque applied perpendicularly to the point mass is therefore $\tau = I\alpha$.

The torque on the particle is equal to the moment of inertia about the rotation axis times the angular

acceleration. We can generalize this equation to a rigid body rotating about a fixed axis.

[Source](#)

Materials

Fidget Spinner
Ruler
Small Mass
Beaker Tongs

Mobile Device
Vernier Video Analysis
Stopwatch
Tape

Procedure

Part 1: Understanding the Motion of the Fidget Spinner

1. Hold the fidget spinner horizontally. To understand the motion of the fidget spinner, spin the spinner and record the time it takes for the spinner to come to a stop. Record your data in the data table.
2. Repeat for four additional trials.
3. Hold the fidget spinner vertically. To understand the motion of the fidget spinner, spin the spinner and record the time it takes for the spinner to come to a stop. Record your data in the data table.
4. Repeat for four additional trials.

Part 2: Rotational Inertia of the Fidget Spinner

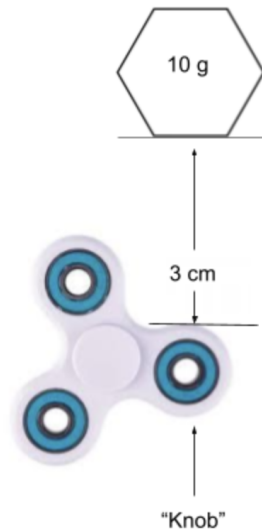
1. Assume that the knob near the outside of the fidget spinner acts as a particle rotating about a fixed axis. Determine the expression for the rotational inertia, I , of one of the knobs in terms of the mass m of the knob and the distance from the center of the knob to the center of the fidget spinner, r .
2. Assume the mass of the fidget spinner is only located in the knobs. Determine the formula for the total rotational inertia of the fidget spinner in terms of I , m , and r .
3. Measure the radius of the fidget spinner, r , from the center of one of the knobs to the center of the spinner and record the value in your data table.
4. Measure the mass of your fidget spinner and record the value in your data table.
5. Calculate the value of the rotational inertia, I , of the fidget spinner in kg m^2 . This is your **actual value for I** . Note: The mass of one knob, m , is $\frac{1}{3}$ the total mass M of the fidget spinner.

Part 3: Rotational Inertia Continued

1. Use a 10 g mass for this part of the experiment.
2. Mark one of the knobs of the fidget spinner with blue tape in order to have a frame of reference for counting the number of revolutions.
3. Note: It may be helpful to use your mobile device to video the rotating motion of your fidget spinner.
4. Note: You may wish to hold the fidget spinner with beaker tongs.

Then,

5. Place the fidget spinner vertically as shown in the image.



6. Place the 10 g mass about 3 from the knob marked with the blue piece of tape.
7. Make sure the fidget spinner is at rest (not moving) and drop the 10 g mass towards the center of the knob.
8. From your video, record the number of rotations the fidget spinner turns after being struck by the 10 g mass.
9. From your video, record the total time, in seconds, for these revolutions.
8. Calculate the period T of the fidget spinner by using the equation $T = \# \text{ revolutions} / \text{total time}$.
9. Calculate the average angular speed, ω_{avg} in rad/sec by using the equation, $\omega_{\text{avg}} = 2\pi/T$.
10. Calculate the net torque $\Sigma\tau$ in Nt m by using the equation $\Sigma\tau = rF \sin \theta$. The force exerted on the fidget spinner (which produces the torque) is the weight of the mass.
11. Calculate the rotational inertia of the fidget spinner in kg m^2 by using

$$\Sigma\tau = I\omega_{\text{avg}}/t$$

Use $t = 0.10$ sec for the time of impact (or use your video to make a valid approximation). This is your **experimental/approximate value** for I .

12. How do your values for rotational inertia compare? Are they close? Or far apart?
13. Should the value of ω_{avg} used in your calculations be an average value or an instantaneous value? Which ω value is larger, instantaneous or average? Explain.
14. How would your calculation for I change if you used an instantaneous value for ω instead of an average value?
15. Calculate the percent error for I using this equation:

$$.\% \text{ error} = \frac{|Approximate - Actual|}{Actual} \times 100)$$

Part 4: Applying Conservation of Energy and Conservation of Angular Momentum

1. How many revolutions do you think the fidget spinner will rotate if you drop the 10 gram mass 6 cm above the knob? Try it!
2. Using conservation of mechanical energy, calculate the translational (linear) speed of the 10 g mass just as it strikes the knob of the fidget spinner when it is released 3 cm and 6 cm above the knob. You should have two answers in m/sec. Assume there is no air resistance.
3. Conservation of angular momentum, which states that angular momentum is conserved unless acted upon by a net external torque, may be represented mathematically by $\Delta L = 0$. As you will learn later,

$$mvr = I\omega$$

Where r is the distance between the center of the 10 g mass to the center of the fidget spinner when the 10 g mass collides with the knob; I is the actual value of the rotational inertia; and v is the linear speed calculated in step 2.

Calculate the instantaneous angular speed ω of the fidget spinner after the 10 g mass strikes the knob of the fidget spinner when it is dropped from 3 cm. Assume that the 10 g mass stops moving after it strikes the knob and there are no external forces.

4. Then calculate the instantaneous angular speed ω of the fidget spinner after the 10 g mass strikes the knob of the fidget spinner when it is dropped from 6 cm. Assume that the 10 g mass stops moving after it strikes the knob and there are no external forces.
5. Is the instantaneous angular speed ω greater than when the 10 g mass is dropped from 6 cm compared to 3 cm? Explain.
6. How does the average angular speed ω_{avg} compare to the instantaneous angular speed ω (when the mass is dropped from a height of 3 cm)? Which value is greater? Why? Which one is most accurate? Explain.
7. Calculate the percent error between the average angular speed ω_{avg} and the instantaneous angular speed (when the mass is dropped from a height of 3 cm).

$$.\% \text{ error} = \frac{|Approximate - Actual|}{Actual} \times 100)$$

8. Why do you think you were asked to do this comparison?

Data

Table 1

| Trial | Time to Stop: Horizontal Spin (sec) | Time to Stop: Vertical Spin (sec) |
|-------|-------------------------------------|-----------------------------------|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

Table 2

| Variable | Value |
|--|-------|
| Mass of Fidget Spinner (kg) | |
| "Radius" of Fidget Spinner (m) | |
| Rotational Inertia, I (kg m ²) | |

Table 3

| Variable | Value |
|---|-------|
| Number of Rotations | |
| Total time (sec) | |
| Period (T: time/# of revolutions) | |
| Average angular speed, ω_{avg} (rad/sec) | |
| Net torque, $\Sigma\tau$ (Nt m) | |
| Rotational inertia, I (kg m ²) | |
| Percent error | |

Table 4

| Variable | Value |
|--|--------------|
| Instantaneous angular speed, ω (3 cm) | |
| Instantaneous angular speed, ω (6 cm) | |
| Percent Error | |

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