Honors Physics

Simple Harmonic Motion Lab

Introduction

One simple system that vibrates or oscillates is a mass hanging on a spring. The force applied by an ideal spring is proportional to how much it is stretched or compressed. Given this force behavior, the up and down motion of the mass is referred to as simple harmonic motion (since the acceleration of the mass is proportional to the square of the displacement).

The position can be modeled with the equation

$$y = A \sin (2\pi ft + ø)$$

Where y is the vertical displacement from the equilibrium position, A is the amplitude of the motion, f is the frequency of the oscillation, t is the time and ϕ is a phase constant. We can represent these quantities graphically as shown below.

Theory

An ideal spring is a system where the generated force is linearly dependent on how it is stretched. Hooke's Law describes this behavior. In order to extend a spring by an amount Δx from its previous position, one needs a force F which is determined by $F = k\Delta x$. Hooke's Law states that

$$\mathbf{F_s = - k\Delta x}$$

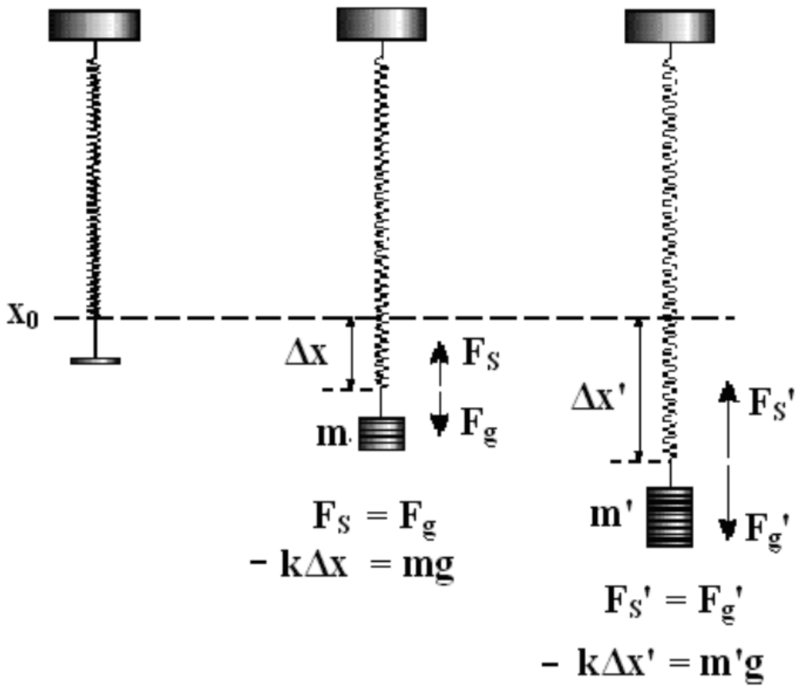
Where k is the spring constant, Fs is the spring force and Δx is the distance the spring is stretched or compressed. The force Fs is a restorative force and its direction is opposite to the direction of the spring's displacement Δx.

To verify Hooke's Law, we must show that the spring force Fs and the distance the spring is stretched Δx are proportional to each other (that is, linearly dependent on each other) and that the constant of proportionality is k.

In this case, the external force is provided by attaching a mass (m) to the end of a spring. The mass will be acted on by the gravitational force, so the force exerted downward on the spring will be Fg = mg. Consider the forces exerted on the attached mass. The gravitational force (mg) points in the -y direction.. The force exerted by the spring ($- k\Delta x$) acts in the +y direction. When the mass is attached to the spring, the spring will stretch until it reaches the point where the two forces are equal but pointing in opposite directions.

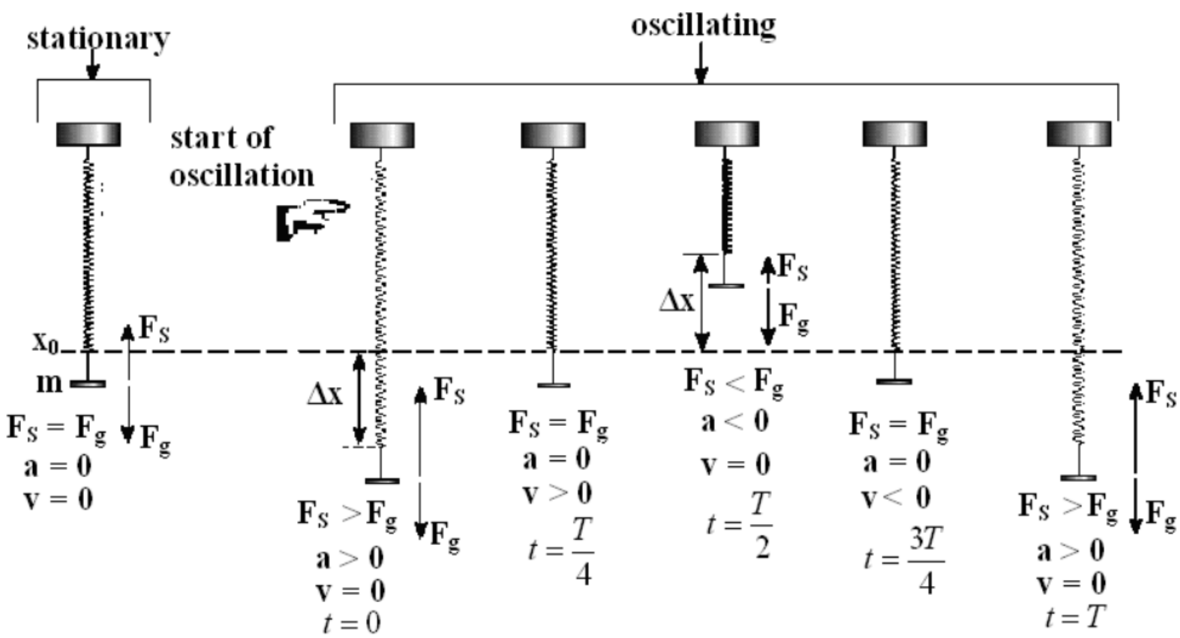
$$F_s - F_g = 0 \text{ or } mg = -k\Delta x$$

The point where the forces balance each other out is known as the equilibrium point. The spring-mass system can stay at the equilibrium point as long as no additional external forces are exerted on the system. The relationship above allows you to determine the spring constant k when m, g, and Δx.



Oscillations

The position where the mass is at rest is called the equilibrium position ($x = x_o$). If the spring is stretched beyond its equilibrium point by pulling it down and then releasing it, the mass will accelerate upward ($a > 0$), because the force due to the spring is greater than the gravitational force pulling it down. The mass will then pass through the equilibrium point and continue to accelerate upward. Once above the equilibrium position, the mass will slow because the net force acting on the mass is now downward. The mass and spring will stop and then the downward acceleration will cause the mass to "move" back down again. As a result, the mass will oscillate around the equilibrium position, as shown below.



The oscillation will proceed with a characteristic period, T, which is determined by the spring constant and the total attached mass. The period is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Where k is the spring constant and m is the hanging mass, assuming the ideal case where the spring itself is massless. For this lab, the spring cannot be treated as massless so you will add ⅓ of its mass to the hanging mass when calculating m,

In order to determine the spring constant, k, from the period of oscillation, T, it is convenient to square both sides of the equation above, giving:

$$T^2 = (4\pi^2/k)\, m$$

This equation has the same form as the equation of a line, y = mx + b, with the y-intercept of zero (b = 0). When plotting T^2 v m, the slope is related to the spring constant by:

Slope = $4\pi^2/k$

So the spring constant can be determined by measuring the period of oscillation for different hanging masses. This is the second way that k will be determined.

In this lab, you will measure the spring constant of a given spring in two ways. First, you will gradually add mass (m) to the spring and measure its displacement (Δx) when in equilibrium. Using Hooke's Law, you will plot Fs v. Δx to find the experimental spring constant. Second, you will measure the spring's period of oscillation, T, for various hanging masses. You will then plot T^2 v m to find the experimental spring constant. You will check whether the two values of k are consistent and if your spring obeys Hooke's Law.

Materials

Meterstick
Set of Masses
Photogate
Spring
Graphical Analysis App

Procedure

Part 1: Hooke's Law

1. Record the mass of the mass hanger in your data table.
2. Measure the rest length of the spring and record it in your data table.
3. Attach the empty mass hanger to the spring and measure the position x_o of the end of the spring.
4. Increase the mass on the end of the spring by 10 g.. Measure the height of the spring after each addition and record it in your data table.
5. Increase the mass by 10 g increments, making sure to measure the and record the height of the end of the spring at each step. You need a total of 5 data points.

Part 2: Period of Oscillation

1. Remove all masses from the mass hanger.
2. Open Graphical Analysis on your laptop.
3. Connect the photogate by bluetooth or by USB.
4. Place the photogate so that the mass hanger passes freely through the photogate.
5. Pull the mass hanger down. Do not stretch the spring more than 5 cm.
6. Click **Collect** on Graphical Analysis.
7. Release the mass hanger and let it oscillate 10 times. Be sure that the oscillation is not wobbly. Determine the time for 10 oscillations.
8. Add 10 g to the mass hanger. Repeat steps 5 and 6.
9. Increase the mass by 10 g increments, letting the mass oscillate ten times.
10. You need a total of 5 data points.

Data

Mass of the spring: ___________ kg

Part 1: Hooke's Law

Trial	**m_{hanger} (kg)**	**$M_{hanger + masses}$ (kg)**	**$F_g = F_s$**

1			
2			
3			
4			
5			

Trial	**x_o (m)**	**x_{total} (m)**	**$\Delta x = x - x_o$**
1			
2			
3			
4			
5			

Part 2: Oscillations

Trial	**m_{hanger} (kg)**	**$M_{hanger + masses}$ (kg)**	**Δm**	**t (10 oscillations)**	**T (sec)**	**T^2 (sec^2)**
1						
2						
3						
4						
5						

Analysis
Part 1: Hooke's Law

1. Calculate Δm and Δx.
2. Calculate the gravitational force acting on the spring $F_G = (\Delta m)g$, which is also equal to Fs. You are using Δm, the amount of mass that was added to the hanger, because you measured the distance the spring stretched (Δx) from the starting point x_o.

2. Graph F_s v. Δx. Fs should be on the y axis (dependent variable) and Δx should be on the x axis (independent variable) in Google Sheets. Be sure to include a title for your graph and be sure that your axes are labeled.
3. Determine the slope of the best fit line. This is your experimental value for k.
4. Look at your R value (correlation coefficient). The closer this value is to 1.000, the better the curve fit.

Part 2: Period of Oscillation
1. Calculate T and T^2.
2. You need to take into account the mass of the spring (as the spring is not considered to be massless). To do this, add one third of the spring's mass to the total hanging mass.
3. Graph T^2 v. m. T^2 should be on the y axis (dependent variable) and m (the total mass) should be on the x axis (independent variable) in Google Sheets. Be sure to include a title for your graph and be sure that your axes are labeled.
4. Determine the slope of the best fit line. You can determine the experimental value for k by using this relationship:

Slope = $4\pi^2/k$

5. Look at your R value (correlation coefficient). The closer this value is to 1.000, the better the curve fit.
6. Create a second graph of T^2 v. m, except assume the spring is massless. That is, do not add the ⅓ of the spring's mass to your total mass.
7. Determine the slope of the best fit line. You can determine the experimental value for k by using this relationship:

Slope = $4\pi^2/k$

8. Look at your R value (correlation coefficient). The closer this value is to 1.000, the better the curve fit.

Questions
1. You will be provided with the theoretical (actual) value of k for your spring. Calculate the percent error for your Hooke's Law analysis and for your period of oscillation analysis.
2. Is your data consistent with Hooke's Law? Specifically, is the spring force linearly dependent on how much the spring is stretched?
3. Discuss the consistency of your two measurements of the spring constant. If they are not consistent, suggest a possible source of error. Hint: Air resistance is not one of them.
4. How significant is the assumption that the spring is massless? You may use your r value (correlation coefficient) as a guide.